Longman's Hill Calculation Guidance



This calculation guidance has been developed and influenced by the NCETM's guidance (2015). At Longman's Hill, we use the NCETM Curriculum Prioritisation materials to plan coherent lessons. The following guidance summarises some of the key features of our lessons. It is not intended to be an exhaustive list, but rather to inform and to enhance the teaching of mathematics across all year groups.

KEY FEATURES OF OUR LESSONS

Developing fluency with basic number facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. It has been found that spending a short amount of time everyday on these facts quickly leads to improved fluency. At Longman's Hill, we follow the Mastering Number Programme from EYFS to Y3 which focuses on additive facts and then in Year 4 and 5, Mastering Number at KS2 which focuses on fluency in times-tables. Alongside choral repetition, and the expectation to recall these facts instantly, children

will develop a deep conceptual understanding using many of the aspects outlined below.

(see Mastering Number progression overview (KS1 and KS2) for more information and a detailed breakdown of year on year development).

At the end of Y1, Y2 and Y3, children will be tested on their appropriate year-group additive facts. This information will then be passed up to the next class teacher to inform the child's next steps.



+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

In Year 4, children will be assessed using Times Table Rock Stars (TTRS), in preparation for the MTC check in June. This information will be passed up to Year 5. Children in Year 5 will continue to access the online learning platform, TTRS, to continually practise the recall of times tables.

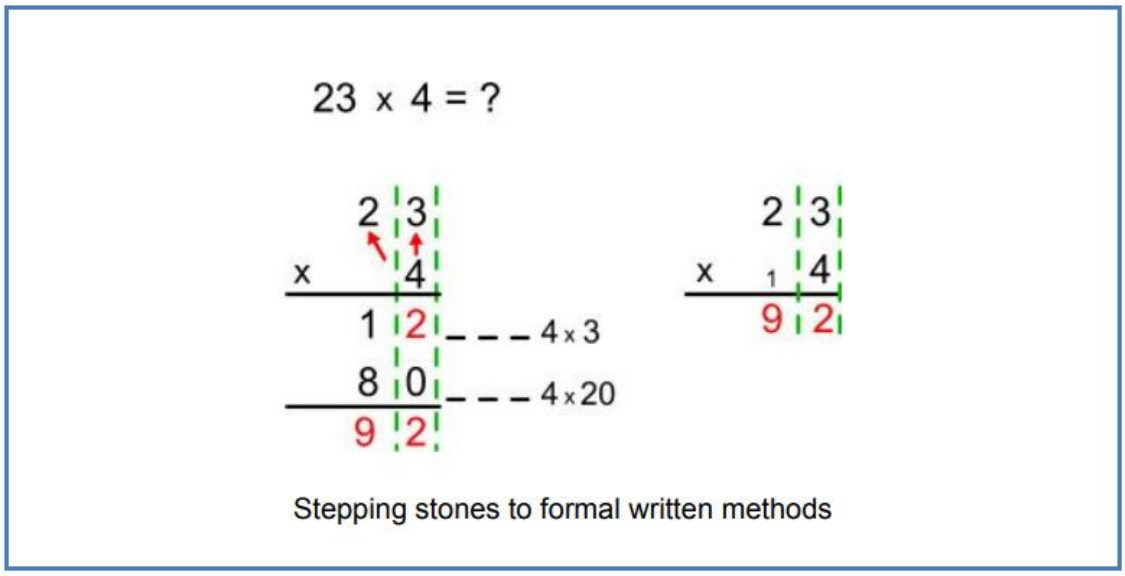
×10	×5	×2	×4	×8	×3	×6	×9	×7

<u>Developing fluency with formal written methods</u>

Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. Building on the well-evidenced

approach to the teaching of maths in Shanghai, we ensure that children understand the structure of the mathematics presented in the algorithms, by placing a particular emphasis on place value. We use base ten equipment (Dienes, place value counters) and images of these, to support the development of fluency and understanding.

Informal methods of recording calculations are an important first stage to help children develop fluency with formal methods of recording. These informal methods are only used for a short period, to help children understand the internal logic of formal methods of recording calculations. These are the stepping stones to formal written methods. Here is an example ...



Develop children's understanding of the = symbol

The symbol = is an assertion of equivalence. If we write:

3 + 4 = 6 + 1

Then we are saying that what is on the left of the = symbol is equivalent to what is on the right of the symbol. We want children to understand this equivalence and not that = means 'the answer'. Many children interpret = as being simply an instruction to evaluate a calculation and then as a result see this:

3 + 4 =

5 X 7 =

16 – 9 =

If children only think of = as meaning 'work out the answer to this calculation' then they are likely to get confused by empty box questions:

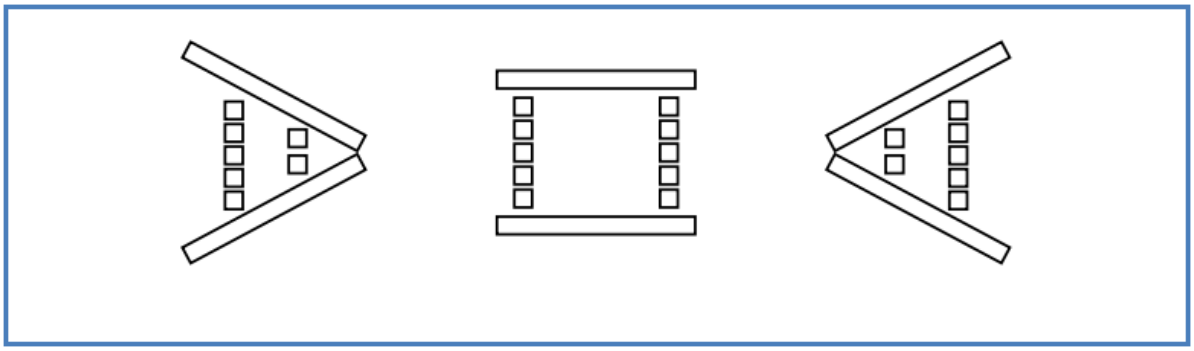
3 + □ = 8, and then later with even simple algebraic calculations such as

3y = 18

To model equivalence, teachers can use balance scales. Also, teachers will vary the position of = symbol and include empty box problems from Year 1 to deepen children's understanding of the = symbol.

<u>Teach inequality alongside teaching equality</u>

To help children develop their understanding of equality, they also need to develop their understanding of inequality. One way to introduce the < > signs is to use rods and cubes (Base 10/Dienes) to make a concrete and visual representation. We use the representation below to represent < > and =



Incorporating both equality and inequality into examples and tasks can help develop children's conceptual understanding. For example like this;

5 + 7 □ 5 + 6

This type of activity will encourage children to develop mathematical reasoning.

"I know that 7 is greater than 6, so 5 plus 7 must be greater than 5 + 6.

Asking children to decide if number statements are true or false also helps to develop reasoning. For example, in discussing this statement;

4 + 6 + 8 > 3 + 7 + 9

A child might reason that "4 plus 6 and 3 plus 7 both equal 10. However, 8 is less than 9, therefore 4 + 6 + 8 must be less than 3 + 7 + 9, not more.

In both of these examples, the numbers have been chosen deliberately to allow children to establish the answer without actually needing to do the computation. This emphasises further the importance of mathematical reasoning.

Don't count, calculate

Young children benefit from being helped at an early stage to start calculating, rather than relying on 'counting on' as a way of calculating. For example, with a sum such as:

4 + 7 =

Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11.

At Longman's Hill, we follow the NCETM's Mastering Number programme in EYFS to Year 3 which aims for children to become automatic with their recall of additive facts. Teaching children to calculate, not count, supports this aim and reduces the child's reliance on counting, which can be an inefficient strategy.

Look for patterns and make connections

The NCETM's visits and research in Shanghai found that teachers used a great many visual representations of the mathematics and some concrete resources. Understanding, however, does not happen automatically; children need to reason by themselves and make their own connections. The Shanghai teachers talked about getting children into good habits from Year 1, in terms of

reasoning and looking for pattern and connections in the mathematics. The question "What's the same, what's different?" is used frequently to make comparisons. For example *"What's the same, what's different between the three times table and the six times table?"*

Use intelligent practice

Children need to engage in a significant amount of practice of mathematics in class. However, *in designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity* (Gu, 1991). The practice that children engage in provides the opportunity to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. The connections they make should improve their fluency.

For example:

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Shanghai Textbook Grade 2 (aged 7/8)

Using empty box problems

Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They provide the opportunity for reasoning and finding easy ways to calculate. They enable children to practise procedures, whilst at the same time thinking about conceptual connections.

A sequence of examples such as

$$3 + \square = 8$$

$$3 + \square = 9$$

$$3 + \square = 10$$

$$3 + \square = 11$$

helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the missing addend.

This sequence of examples does the same at a deeper level:

$$\mathbf{3 \times \square + 2 = 20}$$

$$\mathbf{3 \times \square + 2 = 23}$$

$$\mathbf{3 \times \square + 2 = 26}$$

$$\mathbf{3 \times \square + 2 = 29}$$

$$\mathbf{3 \times \square + 2 = 35}$$

Children should also be given examples where the empty box represents the operation, for example:

4 × 5 = 10 □ 10

6 □ 5 = 15 + 15

6 □ 5 = 20 □ 10

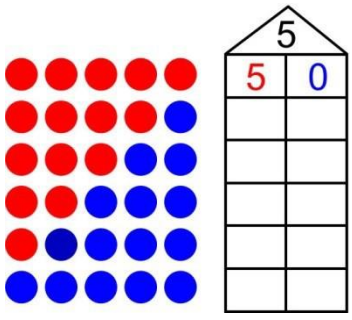
8 □ 5 = 20 □ 20

8 □ 5 = 60 □ 20

These examples also illustrate the careful use of variation to help children develop both procedural and conceptual fluency.

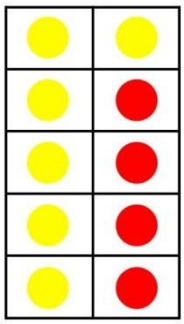
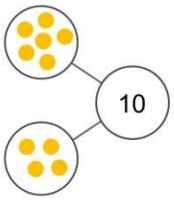
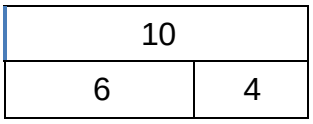
Expose mathematical structure and work systematically

Developing instant recall alongside conceptual understanding of number bonds to 10 is important. This can be supported through the use of images such as the example illustrated below:



The image lends itself to seeing pattern and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5.

Using other structured models such as tens frames, part whole models or bar models can help children to reason about mathematical relationships.

 6 + 4 = 10 4 + 6 = 10 10 – 4 = 6 10 – 6 = 4	 6 + 4 = 10 4 + 6 = 10 10 – 4 = 6 10 – 6 = 4	 6 + 4 = 10 4 + 6 = 10 10 – 4 = 6 10 – 6 = 4
Tens Frame	Part Whole Model	Bar Model

Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question "What's the same, what's different?" has the potential for children to draw out the connections.

Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same - it is only the numbers that change.

For example:

10 6 | 4	247 173 | 74	6.2 3.4 | 2.8
6 + 4 = 10 4 + 6 = 10 10 – 6 = 4 10 – 4 = 6	173 + 74 = 247 74 + 173 = 247 247 – 173 = 74 247 – 74 = 173	3.4 + 2.8 = 6.2 2.8 + 3.4 = 6.2 6.2 – 3.4 = 2.8 6.2 – 2.8 = 3.4

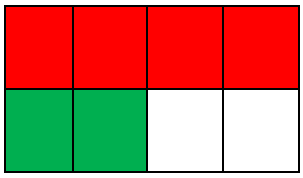
Move between the concrete and the abstract

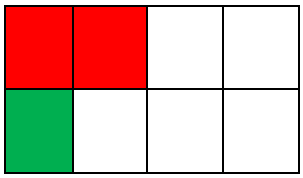
Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols.

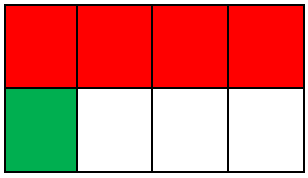
For example, in a lesson about addition of fractions, children could be asked to draw a picture to represent

$$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

Alternatively, or in a subsequent lesson, they could be asked to discuss which of three visual images correctly represents the sum, and to explain their reasoning:







Contextualise the mathematics

A lesson about addition and subtraction could start with this contextual story:

"There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are now on the bus?"

This helps children develop their understanding of the concepts of addition and subtraction. During the lesson, the teacher should keep returning to the story. For example, if the children are thinking about this calculation;

$$11 - 6 =$$

then the teacher should ask the children:

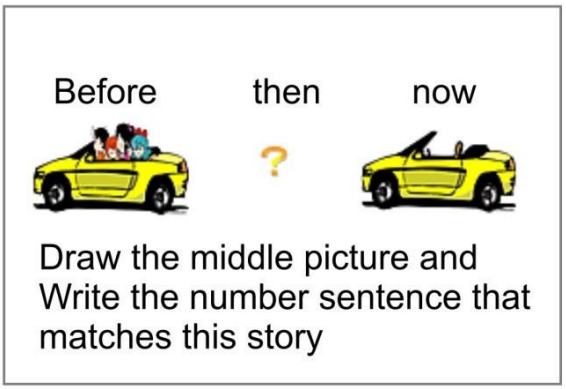
"What does the 11 mean? What does the 6 mean?", expecting that children will answer:
"There were 11 people on the bus, and 6 is the number who got off."

Then asking the children to interpret the meaning of the terms in a sum such as 7 + 7 = 14 will give a good assessment of the depth of their conceptual understanding and their ability to link the concrete and abstract representations of mathematics.

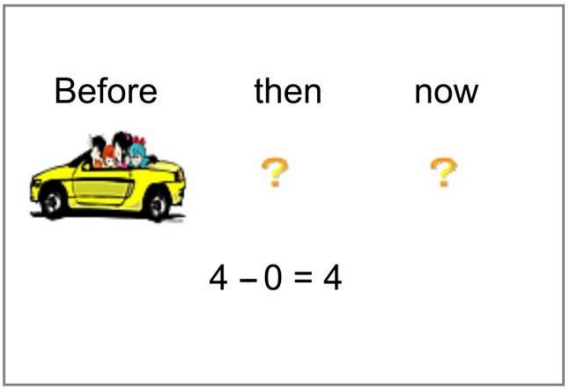
The four slides below are taken from a lesson delivered by one of the Shanghai teachers (Li Dong)



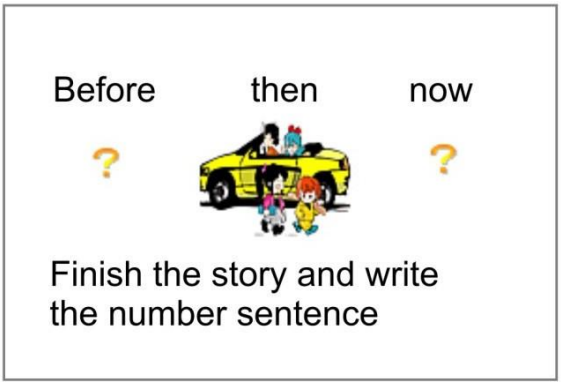
Slide 1



Slide 2



Slide 3



Slide 4

Notice how each activity varies. The children are asked to:

Slide 1: Start with the story (concrete) and write the number sentence (abstract).

Slide 2: Start with the story (concrete) and complete it. Then write the number sentence (abstract).

Slide 3: Start with the number sentence (abstract) and complete the story (concrete).

Slide 4: Start with part of the story, complete two elements of it (concrete with challenge) and then write the number sentence (abstract).

The children move between the concrete and the abstract and back to the concrete, with an increasing level of difficulty.

Use questioning to develop mathematical reasoning

Teachers' questions in mathematics lessons are often asked in order to find out whether children can give the right answer to a calculation or a problem. However, in order to develop children's conceptual understanding and fluency there needs to be a strong and consistent focus on questioning that encourages and develops their mathematical reasoning.

This can be done simply by asking children to explain how they worked out a calculation or solved a problem, and to compare and contrast different methods that are described. Teachers should have expectations of children to justify their mathematical reasoning.

Some calculation strategies are more efficient and teachers should scaffold children's thinking to guide them to the most efficient methods, whilst at the same time valuing their own ideas.

Rich questioning strategies include:

- *"What's the same, what's different?"*

In this sequence of expressions, what stays the same each time and what's different?

23 + 10	23 + 20	23 + 30	23 + 40

Discussion of the variation in these examples can help children to identify the relationship between the calculations and hence to use the pattern to calculate the answers.

- *"Odd one out"*

Which is the odd one out in this list of numbers: 24, 15, 16 and 22?

This encourages children to apply their existing conceptual understanding. Possible answers could be:

> *"15 is the odd one out because it's the only odd number in the list." "16 is the odd one out because it's the only square number in the list."*
>
> *"22 is the odd one out because it's the only number in the list with exactly four factors."*

If children are asked to identify an 'odd one out' in this list of products:

24×3 36×4 13×5 32×2

they might suggest:

> *"36 × 4 is the only product whose answer is greater than 100." "13 × 5 is the only product whose answer is an odd number."*

- *"Here's the answer. What could the question have been?"*

Children are asked to suggest possible questions that have a given answer. For example, in a lesson about addition of fractions, children could be asked to suggest possible ways to complete this sum:

$$\square + \square = \frac{3}{4}$$

- *Identify the correct question*

Here children are required to select the correct question:

A 3.5m plank of wood weighs 4.2 kg

The calculation was:

$3.5 \div 4.2$

Was the question:

a. *How heavy is 1m of wood?*

b. *How long is 1kg of wood?*

- *True or False*

  Children are given a series of equations and are asked whether they are true or false:

$4 \times 6 = 23$ $4 \times 6 = 6 \times 4$ $12 \div 2 = 24 \div 4$ $12 \times 2 = 24 \times 4$

Children are expected to reason about the relationships within the calculations rather than calculate.

- *Greater than, less than or equal to (>, <, or =)*

$3.4 \times 1.2 \bigcirc 3.4$ $5.76 \bigcirc 5.76 \div 0.4$ $4.69 \times 0.1 \bigcirc 4.69 \div 10$

These types of questions are further examples of intelligent practice, where conceptual understanding is developed alongside the development of procedural fluency. They also give pupils who are,

"rapid graspers" the opportunity to apply their understanding in more complex ways.

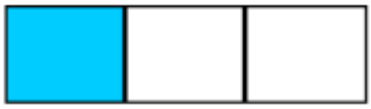
Use the correct mathematical terminology and use complete sentences to express their reasoning

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology (e.g. saying 'digit' rather than 'number') and to explain their mathematical thinking in complete sentences.

I say, you say, you say, you say, we all say

This technique enables the teacher to provide a sentence stem for children to communicate their ideas with mathematical precision and clarity. These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding. For example:

If the rectangle is the whole, the shaded part is one third of the whole.



Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repeated use helps to embed key conceptual knowledge.

Another example is where children fill in the missing parts of a sentence; varying the parts but keeping the sentence stem the same. For example:

There are 12 **stars**. $\frac{1}{3}$ of the **stars** is equal to **4 stars**

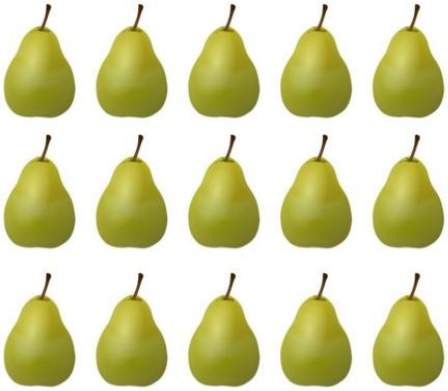


Children use the same sentence stem to express other relationships. For example:

There are 12 **stars**. $\frac{1}{4}$ of the **stars** is equal to **3 stars**

There are 12 **stars**. $\frac{1}{2}$ of the **stars** is equal to **6 stars**

Similarly:



There are **15 pears**. $\frac{1}{3}$ of the **pears** is equal to **5 pears**

There are **15 pears**. $\frac{1}{5}$ of the **pears** is equal to **3 pears**

When talking about fractions it is important to make reference to the whole and the part of the whole in the same sentence. The above examples help children to get into the habit of doing so. Another example is where a mathematical generalisation or "rule" emerges within a lesson. For example:

When adding 10 to a number, the ones digit stays the same.

This is repeated in chorus using the same sentence, which helps to embed the concept.

Identify difficult points

Difficult points need to be identified and anticipated when lessons are being designed, and these need to be an explicit part of the teaching, rather than the teacher just responding to children's difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties, because if one child has a difficulty, it is likely that others will have a similar difficulty. Difficult points also give an opportunity to reinforce that we learn most by working on, and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding. For example:

$$\frac{2}{14} - \frac{1}{7} = \frac{1}{7}$$

A visualiser is a valuable resource since it allows the teacher quickly to share a child's thinking with the whole class.